Measurements of Fluctuating Temperatures in a Supersonic Turbulent Flow Using Laser-Induced Fluorescence

Kenneth P. Gross*

Polyatomics Research Institute, Mountain View, California and

Robert L. McKenzie†

NASA Ames Research Center, Moffett Field, California

A laser-induced fluorescence technique that provides a practical means of nonintrusively measuring the instantaneous temperatures in low-temperature turbulent flows has been developed. It relies on the fluorescence from small concentrations of NO in a flow of N_2 . The technique has been demonstrated in a two-dimensional turbulent boundary layer at Mach 2. Both one and two-photon excitation methods were used, but limits imposed on the two-photon method by Stark effects rendered it less accurate. Single-pulse measurements, in flows with 100 ppm NO, were obtained in the range from 150 to 295 K with an uncertainty of 1% rms. Fluctuations up to 6% rms were observed, owing to the turbulent flow. The average temperature distribution through the boundary layer agreed well with temperatures implied by pitot-probe data.

Introduction

A present, few nonintrusive diagnostic methods have been demonstrated that have sufficient spatial and temporal resolution for accurately measuring the fluid-dynamic parameters of interest in low-temperature, compressible, turbulent flows. Except for velocity measurements using well-established laser-Doppler-velocimetry methods, most reported optical techniques are unable to resolve the turbulent fluctuations. Quantities such as temperature, density, and their correlation with velocity remain as important variables to be obtained for the experimental verification of turbulence models.

Time-resolved light-scattering techniques using Rayleigh or spontaneous Raman processes have been applied to the measurement of temperature and density, 1-3 but the observation of turbulent fluctuations is typically made difficult by one or more sources of background noise. For the case of Rayleigh scattering,4-6 the signal is at the same frequency as the laser, requiring the careful rejection of stray light. The presence of particulates in the flow can easily confuse the signal analysis. Single-pulse Raman measurements have been reported, 7,8 but the small Raman scattering cross section typical of all common gases leads to very weak signals. Laser pulse energies approaching 1 J are required to achieve the signal-to-noise ratios necessary to resolve fluctuations due to turbulence. Modern nonlinear Raman processes, such as coherent anti-Stokes Raman scattering (CARS),9 have also been applied to turbulent flows, but primarily in high-temperature combustion environments where molecular vibrational temperatures can be used as an indicator of the bulk gas temperature.

In this paper, a method is described that is based on the strong and frequency-shifted signals from laser-induced fluorescence (LIF). The keystone to implementing such a method is the identification of a fluorescent species with spectroscopic features that are resonant with available laser frequencies, and with chemical behavior that is compatible with the operating requirements of an aerodynamic flow facility. Species that are corrosive, reacting, or that condense in an expanding flow are usually impractical in most aerodynamic

research applications. LIF techniques have been reported ¹⁰⁻¹³ that are based on a variety of species with practical application to particular experimental situations, but most use either a transient species, such as OH, that is available only in high-temperature combustion flows, or they rely on materials that tend to condense at the low temperatures of most supersonic wind tunnel flows.

An excellent candidate for use in aerodynamic research is nitric oxide (NO). Its chemical stability, in combination with strong spectroscopic features that are accessible by commercially available lasers, make it a practical seed gas for the applications addressed here. Conversely, the gas is toxic, requiring it to be used in very dilute concentrations; and it will oxidize in air, limiting its use in airflows. However, we show that in N_2 , it allows nonintrusive measurements of temperature in flows of interest that are not now possible by any other means.

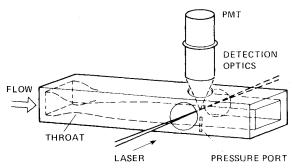
The sensitive measurement of temperature in turbulent and low-temperature flows requires several further considerations regarding the spectroscopic attributes of the fluorescent species. Those of primary importance include: 1) the presence of absorbing and fluorescing transitions with sufficient strength to provide low-noise signals, 2) spectral features that can be resolved with sufficient detail to accurately infer a temperature, and 3) the availability of sufficient information to accurately account for the effects of fundamental molecular processes affecting the fluorescence signals, such as collisional quenching and transition spectral broadening. Most of these processes are dependent on temperature and density, and their variation in a turbulent flow must be included in the signal analysis. In addition, to obtain accurate spectroscopic measurements of temperatures below 500 K, the use of resolved rotational line spectra is most often required because only rotational energy differences are smaller than, or comparable to, the thermal energy being measured. An attractive feature of NO is that it is a diatomic molecule, with simply analyzed rotational spectra, that satisfies all of the preceding requirements.

The measurement technique applied here is similar to that described previously. 14,15 It relies on the detection of broadband ultraviolet (uv) fluorescence following laser excitation of two transitions originating from different rotational states in the $NO(X^2\pi, v''=0 \mapsto A^2\Sigma^+, v'=0)$ vibronic band. Since the fluorescence energy following each excitation is proportional to the energy absorbed by the gas for each transition, and that in turn is proportional to the initial-state number densities, the ratio of fluorescence energies for the two transitions can be

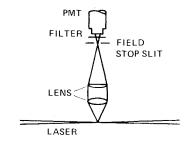
Presented as Paper 84-1536 at the AIAA 17th Fluid/Plasma Dynamics and Lasers Conference, Snowmass, CO, June 25-27, 1984; received July 19, 1984; revision received Feb. 14, 1985. This paper is declared a work of the U.S. Government and therefore is in the public domain.

^{*}Research Scientist.

[†]Research Scientist. Member AIAA.



 a) Test section depicting laser access and fluorescence emission ports.



b) View of light collection geometry and imaging system.

Fig. 1 Boundary-layer channel.

related to the rotational temperature of the ground-state NO molecules through the Boltzmann equation. In its simplest form, the relation is

$$\frac{S_2}{S_1} \sim \frac{n_2}{n_1} \sim \exp\left[\frac{-(E_2 - E_1)}{kT}\right]$$
 (1)

where S_i is the time integral of the fluorescence intensity from transitional i, n_i the initial-state number density, E_i the initial-state rotational energy, k the Boltzmann constant, and T the rotational temperature. The rotational temperature is assumed to be closely coupled to the kinetic temperature of the gas mixture so that the two remain equal for all flow conditions of practical interest.

Equation (1) is used in conjunction with the assumption that all radiative and nonradiative decay rates of the upper states, populated by each laser excitation or by the redistribution of energy among upper states prior to decay, are equal. For the upper electronic state of $NO(A^2\Sigma^+, v'=0)$, quenching rates and fluorescence lifetimes are known to be insensitive to rotational quantum number 16,17 and, hence, are nearly equal for all rotational transitions observed in the broadband fluorescence emission. Consequently, the degree of rotational redistribution preceding the emission is not important if fluorescence is observed from all rotational transitions in a given vibronic band. Since the rates are nearly equal, their ratio is near unity and they have no influence on Eq. (1).

Fluorescence excitation may be accomplished at laser frequencies that are resonant for either single or two-photon absorption. In the latter case, the molecules absorb two photons with frequencies at half the transition frequency. The two rotational transitions necessary for Eq. (1) are excited using two independent, tunable, lasers. The pulses from each laser are separated slightly in time, allowing the fluorescence from both to be recorded simultaneously with a single detection system. Subsequent deconvolution of the double-pulse waveforms then leads to the ratio of fluorescence energies. Measurements are made simultaneously in both the turbulent wind tunnel flow and in a nonflowing reference cell at known temperature and pressure. The reference cell signals are used to normalize the variations in wind tunnel signals, owing to pulse-to-pulse fluctuations in laser energy and spectral frequency.

Experimental Method

Wind Tunnel

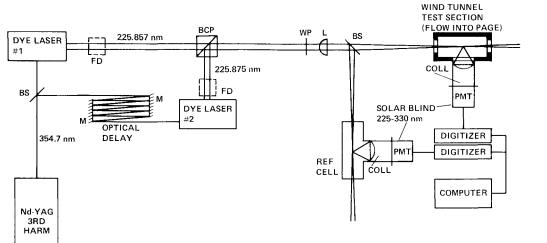
The flow facility used for these experiments is a small blowdown wind tunnel consisting of a high-pressure storage reservoir, a supersonic nozzle and test section, and an evacuated dump tank. The facility can handle toxic gases such as NO at low concentration levels. Up to 300 ppm NO in nitrogen was used for the two-photon LIF measurements. The boundary layer studied was produced on the lower wall of a rectangular Mach 2 channel. Figure 1 shows a simple schematic of the boundary layer channel and depicts the testsection instrumentation ports and collection optics geometry used for the fluorescence measurements. The nozzle has a 25×64 -mm exit, followed by a slightly diverging channel that extends 762 mm from the nozzle throat to the measurement ports. The facility was operated at ambient total temperature with stagnation pressures ranging from 3 to 7 atm. At a stagnation pressure of 3.5 atm, the test section contained a fully turbulent boundary layer filling the upper and lower thirds of its 33-mm height with a laminar core flow in the center. The test time varied between 5 and 20 s, with the upper limit determined by reservoir stagnation pressure and dumptank capacity. Optical access to the flow for the lasers was obtained through 55-mm-diam fused-silica windows mounted on each side of the channel. Fluorescence was observed through a similar window on the top of the channel. Static pressure measurements were made at the channel wall and a pitot tube was temporarily inserted through the top port in the test section to make extensive measurements of total pressure through the boundary layer. Shadowgraphs of the flow taken with and without the pitot tube in place indicated that the measurement volume was free from shock waves or other significant disturbances. The nominal test conditions were as follows: stagnation pressure, $P_t = 2.7$ atm; Mach number, $M_{\infty} = 2.06$; stagnation temperature, $T_1 = 292$ K; freestream temperature, $T_{\infty} = 158$ K; boundary layer thickness, $\delta = 1.2$ cm; and Reynolds number based on δ , $Re_{\delta} = 5 \times 10^5$.

LIF Experiment

The optical arrangement used for the LIF measurements is illustrated in Fig. 2. Two grating-tuned dye lasers were pumped at 10 Hz by the 355-nm third-harmonic output of an Nd:YAG laser. The portion of the pump beam directed to the second dye laser was optically delayed, using two plane mirrors, giving a temporal separation of 125 ns between the two 5-ns dye-laser pulses. Each laser pulse was linearly polarized, and had an energy of a few millijoules contained in a spectral bandwidth of 0.2-0.3 cm⁻¹.

For two-photon LIF measurements, the first and second dye-laser pulses, at wavelengths near 450 nm, were tuned to half the transition frequencies corresponding to the $J''=19\frac{1}{2}$ and $7\frac{1}{2}$, $S_{11}+R_{21}$, two-photon transitions, respectively. For single-photon measurements, each dye-laser output was frequency-doubled with potassium pentaborate (KB5) to produce uv energies of several microjoules. The single-photon transitions used were the $J''=18\frac{1}{2}$, $Q_{11}+P_{21}$ line and the $J''=7\frac{1}{2}$, R_{21} line. In both cases the orthogonally polarized beams were made collinear, focused by a common lens with a 500-mm focal length, and partitioned in the wind tunnel test section and reference cell. The focal spot size at the measuring point was 0.5 mm.

Broadband nitric oxide fluorescence in the spectral range from 225 to 330 nm was collected from the flow channel and reference cell with nearly identical f/1 fused-silica optics. It was transmitted through broadband uv filters (Schott UG-5) and imaged through an aperture that limited the observed sample volume to a 1-mm path length along the laser beam axis. Although reabsorption of the $\gamma(0,0)$ emission was negligible in our experiments, the spectral cutoff of the filters blocked this band, thereby eliminating the influence of reabsorption on the temperature measurement.



BCP: beamcombining polarizer cube. WP: half-wave plate. L: lens.

BS: beamsplitter. M: mirror.

FD: frequency doubler (single-photon only). COLL: collection system. PMT: photomultiplier tube.

Fig. 2 Schematic of the experimental arrangement.

The fluorescence waveforms were detected by solar-blind photomultipliers with peak responsivity in the ultraviolet, and recorded by Tektronix 7912AD transient digitizers interfaced to an HP-1000 computer. The fluorescence recording time was 500 ns. During this time interval the interaction volume is convected downstream approximately 0.25 mm by the Mach 2 flow, and remained within the field of view of the imaging and detection system (Fig. 1b).

The effective temporal resolution for each instantaneous temperature measurement was determined by the 125-ns separation of dye-laser pulses. Spatial resolution was determined by the nearly cylindrical sample volume of approximately 0.5 mm in diameter by 1.0 mm in length.

Before each wind tunnel run, static fluorescence measurements were performed with the flow channel filled with the test gas mixture at ambient temperatures. These prerun tests served to calibrate the measurement system, using the known pressures and temperatures in both the flow channel and reference cell. Gas pressure in the room-temperature reference cell was adjusted so that Voigt line-shape profiles were matched to those in the wind tunnel flow. The Voigt parameters were calculated using the data of Dodge et al. ¹⁸ Line-shape matching was accomplished by equating the normalized Voigt functions at the line center. This procedure maximized the effectiveness for normalization of pulse-to-pulse variations in fluorescence signal amplitude due to the unavoidable frequency jitter of each dye laser.

As described in Ref. 14, the data analysis leading to a temperature value for each pump laser pulse requires knowing the ratio of broadband fluorescence energies resulting from each excitation. The ratio is obtained from each double-pulse waveform by fitting it to a six-parameter function derived for an exponentially decaying emitter driven by a short excitation pulse with a Gaussian temporal profile. The two pulses in each wavefore are then deconvolved and their individual integrals computed. The integral of each pulse is assumed to be linearly proportional to the total fluorescence energy resulting solely from its corresponding laser excitation, with account taken of the laser spectral width and all collision-broadened molecular transitions falling within the excitation bandwidth.

Figure 3 shows an example of some experimental fluorescence waveforms and their functional fits for typical wind tunnel conditions using single-photon excitation. The first pulse results from excitation of the $J'' = 18\frac{1}{2}$ rotational level, and the second pulse is from excitation of the $J'' = 7\frac{1}{2}$ level. Ultraviolet laser energies were in the range of 1-3 μ J for each excitation. For an NO concentration of 100 ppm (Fig. 3a), the photon-statistical noise is very low and the signal-to-noise ratios for each pulse were approximately several hundred to one. For the weaker signals at 5 ppm (Fig. 3b), the photon-statistical noise on the waveforms is evident and easily

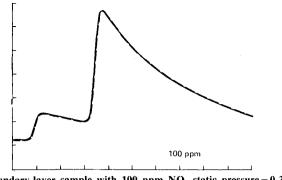
distinguished from the computer-fitted function. The experimental waveforms observed using two-photon excitation and 300 ppm NO had signal-to-noise ratios of about 25, also limited by photon-statistical noise.

Results

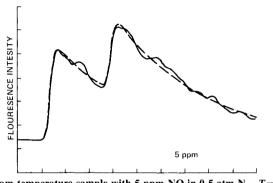
The instrumental uncertainty for the LIF measurements was initially evaluated in a low-temperature static test cell using two-photon excitation. ¹⁵ Generally, the average spectroscopic temperatures agreed to within $\pm 2\%$ with a thermocouple mounted in the cell whereas the instantaneous single-shot temperatures varied between 3 and 4% rms for each data set, thus defining the instrumental rms noise sensitivity using two-photon excitation.

Two-photon excitation of the NO $\gamma(0,0)$ band, although easier and more convenient to implement experimentally than single-photon excitation, suffers from Stark broadening caused by the intense laser fields needed to induce appreciable absorption. 15,19 This effect, which manifests itself as a power dependent broadening of the spectroscopic lines used for the measurement, defeats the use of normalization by a reference cell, and forces the measurements to be made using a minimum laser intensity with a corresponding decrease in signal-to-noise ratio. It is this basic limitation for the twophoton process that limits the temperature measurement noise to approximately 4%. By using single-photon excitation, much higher signal levels can be achieved with much lower laser intensities, giving the added advantage that Stark effects are completely negligible. The primary disadvantages accompanying the use of one-photon excitation are the need for a more complicated experimental setup and the rejection of scattered laser light at wavelengths close to those of the fluorescence signals. In larger scale flows, laser beam absorption along its path to the sample volume may also be troublesome. Fortunately, for NO $\gamma(0,0)$ band excitation and nitric oxide concentrations of 100 ppm or less, the optical transmission through the sample to the measurement point in these experiments was greater than 99%. In addition, the single-photon-induced fluorescence signals were much larger than the stray-light levels encountered, allowing adequate rejection of radiation at the excitation frequency with simple bandpass filters.

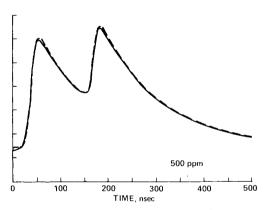
The higher signal levels produced using single-photon excitation also allow a considerable reduction in nitric oxide seeding concentration. Figure 4 is a plot of temperature sensitivity vs NO concentration, using uv laser energies of a few microjoules. It shows that for nitric oxide concentrations above 50 ppm in 0.5-atm N_2 , the rms noise in the temperature measurement is still about 1%. At concentrations below 50 ppm, the effects of photon-statistical noise begin to increase.



a) Boundary-layer sample with 100 ppm NO, static pressure = 0.35 atm, T = 172 K.



b) Room-temperature sample with 5 ppm NO in 0.5 atm N_2 , T = 294



c) Room-temperature reference cell with 500 ppm NO in 0.5 atm N_2 , T=295 K. First pulse is fluorescence from the one-photon $Q_{11}+P_{21}$ (18½) excitation; second pulse is from the R_{21} (7½) excitation. Vertical amplifier bandwidth, 20 MHz; sweep, 50 ns/division.

Fig.3 Dual-pulse fluorescence waveform (solid line) and computerfitted function (dashed line).

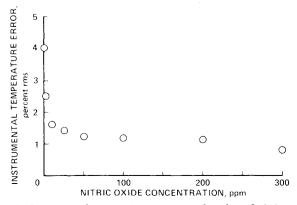


Fig. 4 Instrumental temperature error as a function of nitric oxide concentration, using single-photon excitation in a nonflowing cell at ambient temperature: P = 0.5 atm.

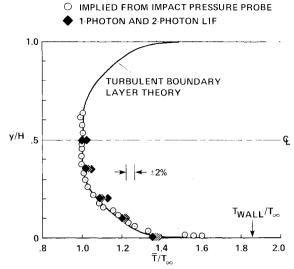


Fig. 5 Average temperature distribution through the boundary layer: channel height H=32.5 mm; $M_{\infty}=2.06$; $T_t=292$ K; $T_{\infty}=158$ K.

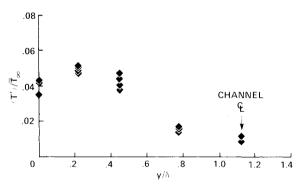


Fig. 6 Distribution of rms temperature fluctuations in the boundary-layer using single-photon LIF; y/δ is the location perpendicular to the wall normalized to the boundary-layer thickness. Gas mixture, 100 ppm NO; $T_{\infty}=158$ (Mach 2); $\delta=1.2$ cm.

Nevertheless, useful measurements can still be made at concentrations of only a few ppm, although interference from scattered laser light will become a larger fraction of the total signal and may require increased spectral filtering. Single-photon measurements using concentrations of about 1-2 ppm are nearly equivalent in noise sensitivity to the corresponding two-photon measurements made using 300 ppm NO.

The results of the wind tunnel tests are illustrated in Figs. 5 and 6. Figure 5 shows the distribution of average temperature obtained by the LIF method as the measurement point is traversed from the channel centerline to a position approximately 0.5 mm from the wall. Both one and two-photon data are presented together and are shown by the solid symbols. They represent averages accumulated during 13-s runs, with account taken of the declining stagnation temperature that is characteristic of blowdown flow facilities. The values plotted are the corresponding average temperatures after 2 s from the start of the run. Since the data acquisition rate was limited by the digitizers to 5 Hz, the averages correspond to 65 temperature determinations per run. The open symbols are temperatures implied from a pitot-probe survey at the same conditions. They are computed using the standard onedimensional equations derived for isentropic compressible flow²⁰ and assuming that the total temperature and static pressure are constant through the boundary layer. The solid theoretical curve is a solution of the compressible, turbulent, boundary-layer equations using a two-layer eddy viscosity model.²¹ Average temperatures determined using either one or two-photon excitation are in agreement with one another and

both agree with the pitot-probe results to within $\pm 2\%$.

Figure 6 depicts the rms magnitudes of the temperature fluctuations due to turbulence, obtained by single-photon LIF for an NO concentration of 100 ppm. The total rms deviations have been corrected by removing the average instrumental noise of 1% determined from the static calibration runs, thus leaving only the fluctuating temperature component resulting from the local flow conditions. Run-to-run variations of the turbulence level determined at the same location in the boundary layer may be due in part to statistical uncertainties resulting from the limited number of data samples taken during a given wind tunnel run.

Conclusion

Ouantitative nonintrusive measurements of fluctuating temperatures have been made using laser-induced fluorescence (LIF) in a simple, well-characterized, Mach 2, turbulent boundary-layer flow seeded with 100-300 ppm of NO. Results obtained using both one and two-photon NO $\gamma(0.0)$ rotational line excitation agree with average temperature profiles deduced from pitot-probe surveys to within 2%. The method using two-photon LIF has been shown to be much less sensitive for instantaneous measurements than the one-photon method: a consequence of optical Stark broadening caused by the high laser field intensities needed for the two-photon measurements. The application of one-photon LIF in this study has demonstrated the capability for directly measuring turbulent fluctuations of temperature as small as 1% of the average local value. In addition, the one-photon technique remains fairly sensitive at much lower NO concentrations, and could be effectively implemented in large scale facilities where lower seeding levels are more desirable.

Acknowledgments

The authors are grateful to D.J. Monson and P. Logan for providing the pitot tube measurements and boundary-layer calculations represented in Fig. 5.

References

¹Peterson, C.W., "A Survey of the Utilitarian Aspects of Advanced Flowfield Diagnostic Techniques," *AIAA Journal*, Vol. 17, Dec. 1979, pp. 1352-1360.

²Lederman, S. and Sacks, S., "Laser Diagnostics for Flowfields, Combustion, and MHD Applications," *AIAA Journal*, Vol. 22, Feb. 1984, pp. 161-173.

³Trolinger, J.D., Azazzy, M., Modarress, D., and Craig, J.E., "Laser Diagnostic Methods/A Summary," AIAA Paper 83-1683, July 1983.

⁴Escoda, M.C. and Long, M.B., "Rayleigh Scattering Measurements of the Gas Concentration Field in Turbulent Jets," *AIAA Journal*, Vol. 21, Jan. 1983, pp. 81-84.

⁵Namer, I., Bill, R.G.Jr., Talbot, L., and Robben, F., "Density Fluctuations in a Flame in a Karman Vortex Sheet," *AIAA Journal*, Vol. 22, May 1984, pp. 647-654.

⁶Gouldin, F.C. and Dandekar, K.V., "Time-Resolved Density Measurements in Premixed Turbulent Flames," *AIAA Journal*, Vol. 22, May 1984, pp. 655-663.

⁷Dibble, R.W., Kollmann, W., and Schefer, R.W., "Conserved Scalar Fluxes Measured in a Turbulent Nonpremixed Flame by Combined Laser Doppler Velocimetry and Laser Raman Scattering," 1982 Fall Meeting of the Western States Section of the Combustion Institute, Sandia National Laboratories, Livermore, CA, Paper WSS/CI 82-52, Oct. 1982.

§Long, M.B., Fourguette, D.C., Escoda, M.C., and Layne, C.B., "Instantaneous Ramanography of a Turbulent Diffusion Flame," *Optics Letters*, Vol. 8, May 1983, pp. 244-246.

⁹Hall, R.J. and Eckbreth, A.C., "Coherent Anti-Stokes Raman Spectroscopy (CARS): Application to Combustion Diagnostics," *Laser Applications*, edited by J.F. Ready and R.K. Erf, Vol. 5, Academic Press, New York, 1984, pp. 214-309.

¹⁰Lucht, R.P., Laurendeau, N.M., Drake, M.C., Lapp, M., and Pitz, R.W., "Single-Pulse, Laser-Saturated Fluorescence Measurements of OH in Turbulent Nonpremixed Flames," *Optic Letters*, Vol. 9, March 1984, pp.90-94.

¹¹McDaniel, J.C., Baganoff, D., and Byer, R.L., "Density Measurement in Compressible Flows Using Off-Resonant Laser-Induced Fluorescence," *Physics of Fluids*, Vol. 25, July 1982, pp. 1105-1107.

¹²Cheng, S., Zimmerman, M., and Miles, R.B., "Supersonic-Nitrogen Flowfield Measurements with the Resonant Doppler Velocimeter," *Applied Physics Letters*, Vol. 43, July 1983, pp. 143-145.

¹³Epstein, A.H., "Quantitative Density Visualization in a Transonic Compressor Rotor," *Engineering for Power*, Vol. 99, July 1977, pp. 460-475.

¹⁴McKenzie, R.L. and Gross, K.P., "Two-Photon Excitation of Nitric Oxide Fluorescence as a Temperature Indicator in Unsteady Gasdynamic Processes," *Applied Optics*, Vol. 20, June 1981, pp. 2153-2165.

¹⁵Gross, K.P. and McKenzie, R.L., "Single-Pulse Gas Thermometry at Low Temperatures Using Two-Photon Laser-Induced Fluorescence in No-N₂ Mixtures," *Optics Letters*, Vol. 8, July 1983, pp. 368-370.

¹⁶Nutt, G.F., Haydon, S.C., and McIntosh, A.I., "Measurement of Electronic Quenching Rates in Nitric Oxide Using Two-Photon Spectroscopy," *Chemical Physics Letters*, Vol. 62, Feb. 1979, pp. 402-404.

 17 McDermid, I.S. and Laudenslager, J.B., "Radiative Lifetimes and Electronic Quenching Rate Constants for Single-Photon-Excited Rotational Levels of NO($A^2\Sigma^+$, v'=0)," Journal of Quantitative Spectroscopy and Radiative Transfer, Vol. 27, May 1982, pp. 483-492.

¹⁸ Dodge, L.G., Dusek, J., and Zabielski, M.F., "Line Broadening and Oscillator Strength Measurements for the Nitric Oxide $\gamma(0,0)$ Band," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 24, Sept. 1980, pp. 237-249.

¹⁹Huo, W.M., Gross, K.P., and McKenzie, R.L., "Optical Stark Effect in the Two-Photon Spectrum of NO," *Physical Review Letters*, Vol. 54, March 1985, pp. 1012-1015.

²⁰"Equations, Tables, and Charts for Compressible Flow," NACA Rept. 1135, 1953.

²¹Cebeci, T. and Smith, A.M.O., Analysis of Turbulent Boundary Layers, Academic Press, New York, 1974.